Aperiodicity in Tree Automata

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- This property corresponds nicely to a class of regular string languages.
- The concept can be extended to tree automata (to tree languages) several ways. One possible extension was introduced by Thomas.
- In this talk we give several other possible definitions of aperiodicity of tree automata, compare them and study them from the algebraic point of view.

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- A tree that is not a variable is called proper.

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- Term functions that can be induced by some proper term are called proper term functions.

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A tree automaton $\mathbb{A}$ is said to be aperiodic iff $C(\mathbb{A})$ is an aperiodic semigroup.

We will denote the class of all aperiodic finite tree automata **CAper**.

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However, when $t \notin T_\Sigma(X_1)$, the induced term function is not a transformation.

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- If $\mathbb{A} = (A, \Sigma)$ is a tree automaton, $n > 0$ is an integer and $t_1, \ldots, t_n \in T_\Sigma(X_n)$ are terms with (at most) n variables, then the tuple $\underline{t} = (t_1, \ldots, t_n)$ induces a term function $\underline{t}^{\mathbb{A}} : A^n \to A^n$ as follows: for any n-tuple of states $\underline{a} = (a_1, \ldots, a_n)$ let

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- Term functions that can be induced by some proper tuple of terms are also called proper.

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- connection to logics;
- a variant of n-aperiodicity has also been introduced and described.

The hierarchy of the classes

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Hence, we can talk about the aperiodicity hierarchy: we have that

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Theorem

For any nonclassical rank type R it holds that the above hierarchy is proper.

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Note that a renaming is not neccessarily reversible: it cannot introduce new function interpretations but can erase some of them.

Generalized cascade product

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Suppose $\mathbb{A} = (A, \Sigma)$ and $\mathbb{B} = (B, \Delta)$ are tree automata and $\alpha = \{\alpha_n : n \in R\}$ is a family of functions such that each α_n maps each element of $A^n \times \Sigma_n$ to some proper n-ary term over Δ.

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Then the generalized cascade product $\mathbb{C} = \mathbb{A} \times_\alpha \mathbb{B}$ is the Σ-tree automaton with carrier set $A \times B$ and where the function symbols are interpreted as follows:

$$\sigma^{\mathbb{C}}\big((a_1, b_1), \ldots, (a_n, b_n)\big) = \big(\sigma^{\mathbb{A}}(a_1, \ldots, a_n), t^{\mathbb{B}}(b_1, \ldots, b_n)\big),$$

where $t = \alpha_n(a_1, \ldots, a_n, \sigma)$.

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There exists a 1-aperiodic regular tree language that is not definable in $\mathrm{FO}(<, S_i)$.

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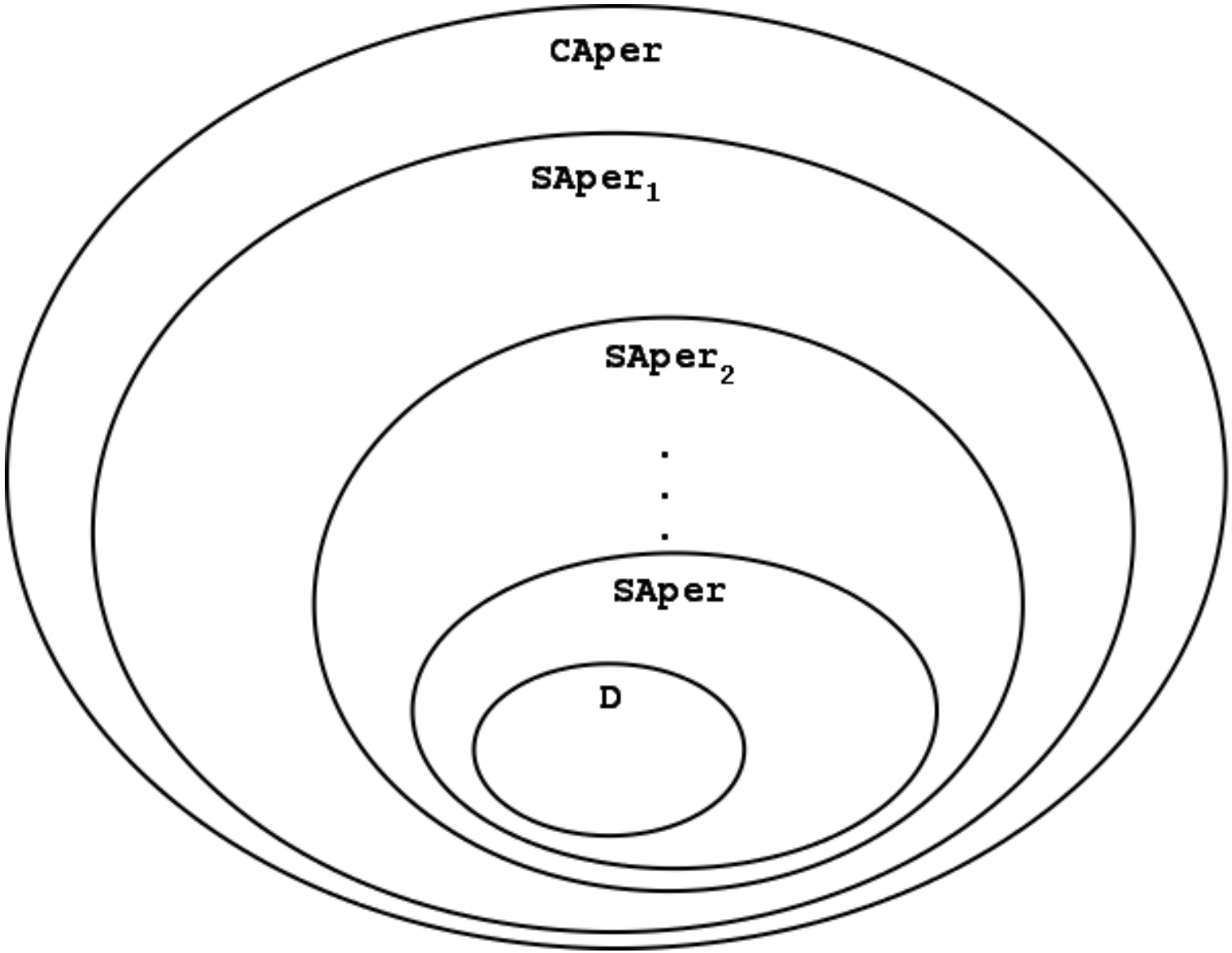
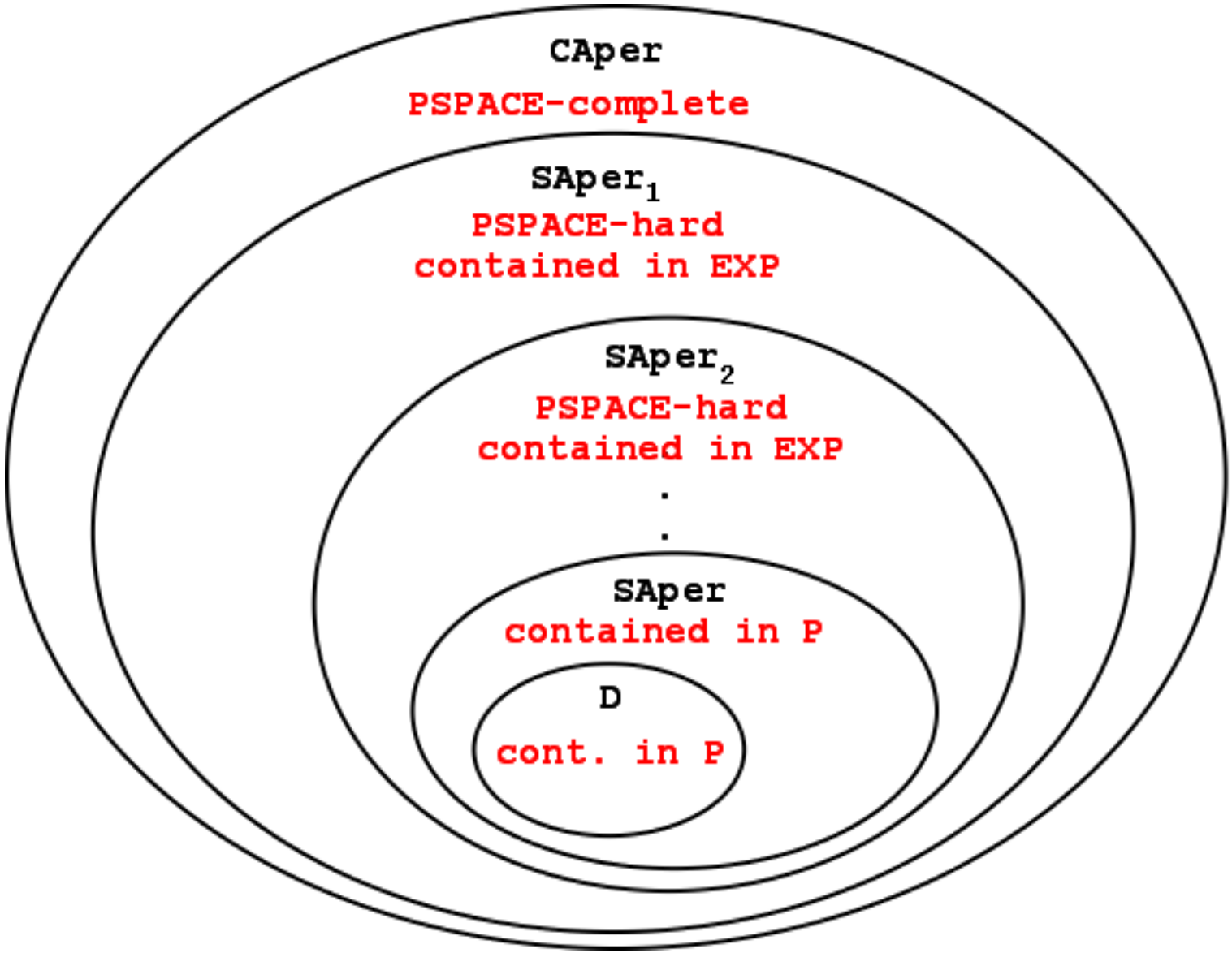
$$\mathbf{D} = \mathbf{SAper}^{(p)} = \mathbf{SAper}_2^{(p)} \subsetneq \mathbf{SAper}_1^{(p)} \subsetneq \mathbf{SAper}_1.$$

(Here $\mathbf{D}$ *denotes the class of all definite automata.)*

The hierarchy



Complexities



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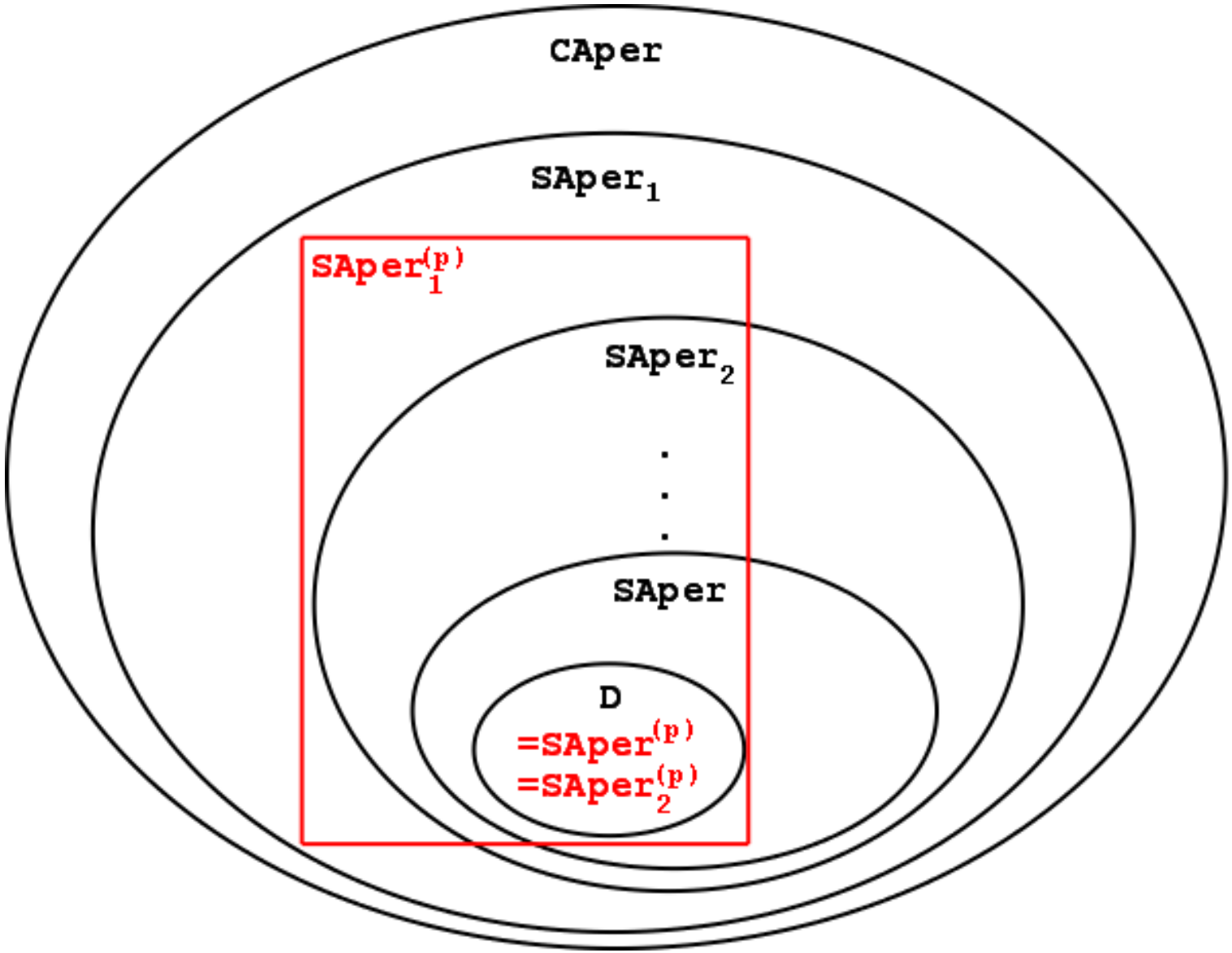
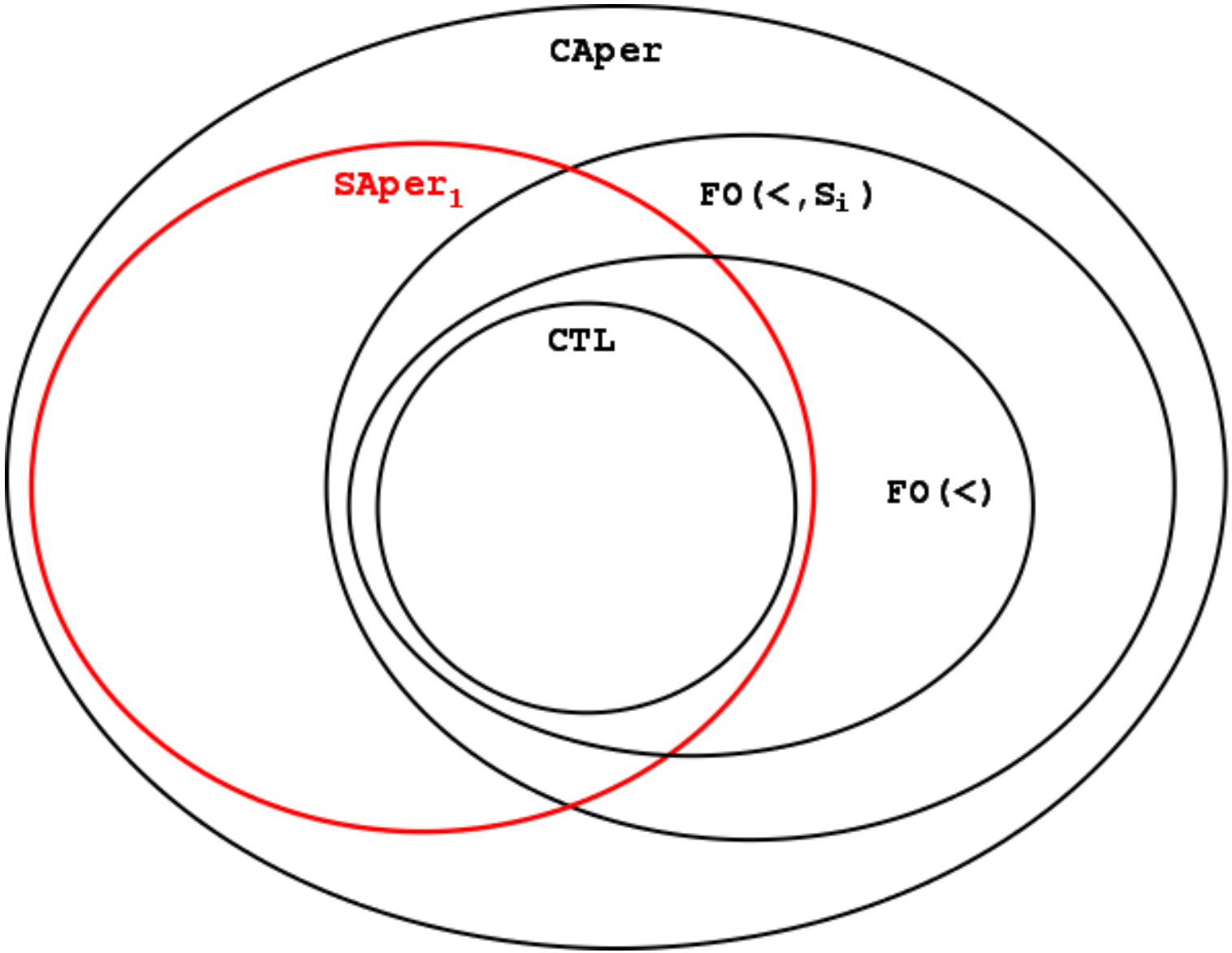


Logics



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- Describe the exact complexity of the membership problem of the classes $\mathbf{SAper}_n$.

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